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A Fractional-Order Approach to Modeling Nutrient–Fish–Mussel Dynamics in Aquatic Ecosystems Using LADM

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Abstract: This study presents an efficient computational approach for solving nonlinear systems of fractional-order differential equations, specifically applied to a fish farm model involving three interacting components: nutrients, fish, and mussels. The proposed method combines the Laplace transform with the Adomian decomposition technique (LADM) to approximate solutions for the Caputo fractional-order system. The Laplace transform helps simplify the differential equations into an algebraic form, while the Adomian decomposition method handles the nonlinear terms systematically. The effectiveness of the LADM is demonstrated through an illustrative example, showcasing its accuracy and applicability in analysing complex ecological systems. This approach provides a powerful tool for researchers studying fractional-order biological and environmental models.

Keywords: Fish farm, Laplace decomposition method, Nonlinear differential equations, Adomian polynomials

MSC: 65L05,92B05

1. Introduction

We consider the following fish farm model [3, 9, 20] with three components: nutrient, fish and mussel

$$\frac{d\eta}{dt} = \omega - (\sigma + \varepsilon F + \xi M)\eta, \frac{dF}{dt} = -(\gamma + \lambda F - \phi\eta)F,$$

$$\frac{dM}{dt} = -(\upsilon - \mu\eta)M, \tag{1}$$

with the initial conditions $\eta(0) = \underline{\eta}, F(0) = \underline{F}, M(0) = \underline{M}$.

Here, the densities of nutrient, fish, and mussel population biomass at time t are denoted as η(t), F(t), and M(t), respectively. The parameters ω, σ, ε, and ξ stand for the maximum nutrient absorption rate of the mussel population, the external food outflow or sedimentation rate of nutrients, and the fish population's nutrient uptake rate, respectively. When the amount of external food increases, the fish population is unable to absorb it, resulting in a eutrophic fish farm. It significantly affects the ecosystem's functioning. The mortality rate, intraspecific competition, and nutrient fraction contributing to fish biomass are denoted by γ, λ, and ϕ, respectively. In this case, υ and μ represent the mussel mortality rate and conversion efficiency, respectively. As a result, the mussels may readily eat the external food that is not incorporated into the fish biomass when it arrives in the form of particle organic matter [8, 9]. In this work, we included the Caputo fractional order derivative in this model (1) in the following way:

$$D^{\alpha_1}\eta = \omega - (\sigma + \varepsilon F + \xi M)\eta, D^{\alpha_2}F = -(\gamma + \lambda F - \phi\eta)F,$$

$$D^{\alpha_3}M = -(\upsilon - \mu\eta)M, \qquad 0 < \alpha_1, \alpha_2, \alpha_2 \leq 1 \qquad (2)$$

with the initial conditions $\eta(0) = \underline{\eta}, F(0) = \underline{F}, M(0) = \underline{M}$. Here $\eta(t), F(t)$ *and* $M(t)$are the densities of nutrient, fish, and mussel population biomass at time t, respectively. Moreover, the parameters $\omega, \sigma, \varepsilon, \xi, \gamma, \lambda, \phi, \upsilon, and \mu$are the same as mentioned above.

2. The Laplace Adomian Decomposition Method

Laplace transforms and the Adomian Decomposition Method (ADM) [1, 2] are combined to create the Laplace Adomian Decomposition Method (LADM). Khuri was the first to introduce this technique [12, 13]. Numerous nonlinear differential equation systems have been solved using LADM, a promising technique [6, 7, 10, 11, 15, 17, 18]. In the Caputo sense, we examine a set of nonlinear fractional-order differential equations of the following form:

$$D^C_{t^{\alpha_i}} y_i(t) = f_i(t, y_1(t), y_2(t), \ldots, y_n(t)), 0 < \alpha_i \leq 1, i = 1,2,\ldots,n$$

with initial conditions:

$y_i(0) = \underline{y_i},$

where $D^C_{t^{\alpha_i}}$ denotes the Caputo fractional derivative of order α_i, and f_i are nonlinear functions of time t and the dependent variables. Taking the Laplace transform of both sides of the equation, and using the property of the Caputo derivative:

Step 1: Apply Laplace Transform

$$L\{D^{C}_{t^{\alpha_i}} y_i(t)\} = s^{\alpha_i} L\{y_i(t)\} - s^{\alpha_i - 1} y_i(0)$$

We obtain:

$$s^{\alpha_i} Y_i(s) - s^{\alpha_i - 1} \underline{y_i} = L\{f_i(t, y_1(t), y_2(t), \ldots, y_n(t))\}$$

Solving for $Y_i(s)$:

$$Y_i(s) = \frac{1}{s} \underline{y_i} + \frac{1}{s^{\alpha_i}} L\{f_i(t, y_1(t), y_2(t), \ldots, y_n(t))\}$$

Step 2: Adomian Decomposition

We assume the solution can be decomposed as:

$$y_i(t) = \sum_{k=0}^{\infty} y_{ik}(t)$$

and the nonlinear term is decomposed into Adomian polynomials:

$$f_i(t, y_1(t), y_2(t), \ldots, y_n(t)) = \sum_{k=0}^{\infty} A_{ik}(t)$$

where A_{ik} are constructed using:

$$A_{ik} = \frac{1}{k!} * \frac{d^k}{d\lambda^k} \left[f_i(t, \sum_{k=0}^{\infty} y_{1k}(t)), \sum_{k=0}^{\infty} y_{2k}(t), \ldots, \sum_{k=0}^{\infty} y_{nk}(t)) \right]_{at\ \lambda = 0}$$

Step 3: Recursive Scheme

Using inverse Laplace transform, we construct the recursive relation:

$$y_{ik+1}(t) = L^{-1}\left\{\frac{1}{s^{\alpha}} * L\{A_{ik}\}\right\}, k \geq 0$$

Step 4: Construct the Approximate Solution

The solution is approximated by truncating the infinite series after a few terms: $y_i(t) \approx \sum_{k=0}^{N} y_{ik}(t)$

The next theorem proves the convergence of the analysis of the Laplace-Adomian decomposition method for nonlinear integro-differential equations [21-23].

Theorem:1 (Convergence of LADM). Consider the nonlinear integro-differential equation (IDE):

$$Lu(x) + Ru(x) + Nu(x) = g(x), \quad x \in [0, T],$$

where: - $L = \frac{d^n}{dx^n}$ is the highest-order derivative operator,

- R is a linear operator (differential or integral),
- N is a nonlinear operator,
- g(x) is a given source term.

Assume the following:

1. Function Space: Let $(B = (C[0,T], \|\cdot\|_\infty)$ be the Banach space of continuous functions on [0,T] equipped with the sup-norm.

2. Lipschitz Conditions:

- The linear operator R satisfies $\|Ru - Rv\|_\infty \le K_R\|u - v\|_\infty$for some $K_R > 0$.

- The nonlinear operator N admits Adomian polynomials $\{A_k\}$ such that $(\|Nu - Nv\|_\infty \le K_N\|u - v\|_\infty)$ for some K_N > 0.

3. Laplace Transform Bound: There exists C > 0 such that: $\|L^{-1}\left[\frac{L[\cdot]}{s^n}\right]\|_\infty \le C\|\cdot\|_\infty$.

4. Contraction Condition: The constants satisfy $\alpha := C(K_R + K_N) < 1$.

Then, the LADM solution $u(x) = \sum_{k=0}^{\infty} u_k(x)$, constructed via the recursive scheme:

$$u_0 = L^{-1}\left[\frac{1}{s^n}\left(\sum_{k=0}^{n-1} s^{n-k-1}u^{(k)}(0) + L[g]\right)\right], \quad u_{k+1} = -L^{-1}\left[\frac{L[Ru_k + A_k]}{s^n}\right],$$

converges uniformly to the unique solution (u^*) of the IDE. Moreover, the error estimate: $\|u^* - u_k\|_\infty \le \frac{\alpha^k}{1-\alpha}\|u_1 - u_0\|_\infty$,holds for all $k \ge 0$.

Proof: Step 1: Reformulation as a Fixed-Point Problem

Apply the Laplace transform to the IDE:

$$L[Lu] + L[Ru] + L[Nu] = L[g].$$

Using the derivative property $L[Lu] = s^n L[u] - \sum_{k=0}^{n-1} s^{n-k-1}u^{(k)}(0)$, we obtain:

$$L[u] = \frac{1}{s^n}\left(\sum_{k=0}^{n-1} s^{n-k-1}u^{(k)}(0) + L[g] - L[Ru + Nu]\right).$$

Define the solution operator $\Phi: B \to B$ by:

$$\Phi u = u_0 - L^{-1}\left[\frac{L[Ru + Nu]}{s^n}\right],$$

where $u_0$is the initial approximation.

Step 2: Contraction Property

For any $u, v \in B$,the difference $\Phi u - \Phi v$ satisfies:

$$\Phi u - \Phi v = -L^{-1}\left[\frac{L[R(u-v) + (Nu - Nv)]}{s^n}\right].$$

Taking norms and applying the Lipschitz conditions:

$$\|\Phi u - \Phi v\|_\infty \leq C(\|R(u-v)\|_\infty + \|Nu - Nv\|_\infty) \leq C(K_R + K_N)\|u - v\|_\infty = \alpha\|u - v\|_\infty.$$

Since $\alpha < 1, \Phi$ is a contraction of B.

Step 3: Banach Fixed-Point Theorem

By the Banach fixed-point theorem, Φhas a unique fixed point $\Phi u^* \in B$satisfying $\Phi u^* = u^*$, which is the solution to the IDE. The iterative scheme $u_{k+1} = \Phi u_k$ converges uniformly to u^*,with the error bound:

$$\|u^* - u_k\|_\infty \leq \frac{\alpha^k}{1-\alpha}\|u_1 - u_0\|_\infty.$$

Step 4: Uniform Convergence of the Series

The LADM series $u(x) = \sum_{k=0}^{\infty} u_k(x)$ is equivalent to the sequence of iterates $\{u_k\}$, whose convergence is guaranteed by the fixed-point theorem.

3. Application of Methodology

The main aim of this section is to discuss the Laplace Adomian Decomposition Method for solving of fish farm model (2). The technique consists first of applying Laplace transformation (denoted throughout this paper by L) to both sides of (2); hence

$$L\{D^{\alpha_1}\eta\} = L\{\omega - (\sigma + \varepsilon F + \xi M)\eta\}, L\{D^{\alpha_2}F\} = L\{-(\gamma + \lambda F - \phi\eta)F\},$$

$$L\{D^{\alpha_3}M\} = L\{-(\upsilon - \mu\eta)M\}, \qquad (3)$$

Applying the formulas for Laplace transforms, we obtain

$$s^{\alpha_1}L\{\eta\} - s^{\alpha_1 - 1}(\eta(0)) = \frac{\omega}{s} - \sigma L\{\eta\} + \varepsilon L\{F\eta\} + \xi L\{M\eta\} s^{\alpha_2}L\{F\} - s^{\alpha_2 - 1}(F(0)) = -\gamma L\{F\} - \lambda L\{F^2\} + \phi L\{\eta F\},$$

$$s^{\alpha_3}L\{M\} - s^{\alpha_3 - 1}(M(0)) = -\upsilon L\{M\} + \mu L\{\eta M\}, \quad (4)$$

Using the initial conditions of the mathematical model (2), the above equations can be reduced to

$$L\{\eta\} = \frac{\eta}{s} + \frac{\omega}{s^{\alpha_1+1}} - \frac{\sigma}{s^{\alpha_1}}L\{\eta\} - \frac{\varepsilon}{s^{\alpha_1}}L\{F\eta\} - \frac{\xi}{s^{\alpha_1}}L\{M\eta\}L\{F\} = \frac{F}{s} - \frac{\gamma}{s^{\alpha_2}}L\{F\} - \frac{\lambda}{s^{\alpha_2}}L\{F^2\} + \frac{\phi}{s^{\alpha_2}}L\{\eta F\},$$

$$L\{M\} = \frac{M}{s} - \frac{\upsilon}{s^{\alpha_3}}L\{M\} + \frac{\mu}{s^{\alpha_3}}L\{\eta M\}. \quad (5)$$

Now, Applying the concept of Adomian Polynomial, we get,

$$L\{\eta\} = \frac{\eta}{s} + \frac{\omega}{s^{\alpha_1+1}} - \frac{\sigma}{s^{\alpha_1}}L\{\eta\} - \frac{\varepsilon}{s^{\alpha_1}}L\{\underline{A}\} - \frac{\xi}{s^{\alpha_1}}L\{\underline{B}\}L\{F\} = \frac{F}{s} - \frac{\gamma}{s^{\alpha_2}}L\{F\} - \frac{\lambda}{s^{\alpha_2}}L\{\underline{C}\} + \frac{\phi}{s^{\alpha_2}}L\{\underline{A}\},$$

$$L\{M\} = \frac{M}{s} - \frac{\upsilon}{s^{\alpha_3}}L\{M\} + \frac{\mu}{s^{\alpha_3}}L\{\underline{B}\}, \quad (6)$$

where, $\underline{A} = F\eta, \underline{B} = M\eta, and \underline{C} = F^2$. The Laplace transform decomposition method consists next of representing the solution as an infinite series, namely

$$\eta(t) = \sum_{k=0}^{\infty} \eta_k, F(t) = \sum_{k=0}^{\infty} F_k and M(t) = \sum_{k=0}^{\infty} M_k \quad (7)$$

where the terms $\eta_k, F_k, and M_k$ are to be recursively computed. Also, the nonlinear operators $\underline{A}, \underline{B} and \underline{C}$ are decomposed as follows:

$$\underline{A}_0 = F_0\eta_0 \underline{A}_1 = F_0\eta_1 + F_1\eta_0 \underline{A}_2 = F_0\eta_2 + F_1\eta_1 + F_0\eta_2$$

$$\underline{A}_3 = F_0\eta_3 + F_1\eta_2 + F_2\eta_1 + F_3\eta_0 \quad (8)$$

$$\underline{B}_0 = M_0\eta_0 \underline{B}_1 = M_0\eta_1 + M_1\eta_0 \underline{B}_2 = M_0\eta_2 + M_1\eta_1 + M_0\eta_2$$

$$\underline{B}_3 = M_0\eta_3 + M_1\eta_2 + M_2\eta_1 + M_3\eta_0 \quad (9)$$

$$\underline{C}_0 = {F_0}^2 \underline{C}_1 = 2F_0F_1 \underline{C}_2 = 2F_0F_1 + {F_1}^2$$

$$\underline{C}_3 = 2F_0F_3 + 2F_1F_2 \quad (10)$$

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Substituting (9) and (10) into (7), we get

$$L\left\{\sum_{k=0}^{\infty} \eta_k\right\} = \frac{\eta}{s} + \frac{\omega}{s^{\alpha_1+1}} - \frac{\sigma}{s^{\alpha_1}} L\left\{\sum_{k=0}^{\infty} \eta_k\right\} - \frac{\varepsilon}{s^{\alpha_1}} L\left\{\sum_{k=0}^{\infty} \underline{A}_k\right\} - \frac{\xi}{s^{\alpha_1}} L\left\{\sum_{k=0}^{\infty} \underline{B}_k\right\} L\left\{\sum_{k=0}^{\infty} F_k\right\} = \frac{F}{s} - \frac{\gamma}{s^{\alpha_2}} L\left\{\sum_{k=0}^{\infty} F_k\right\} - \frac{\lambda}{s^{\alpha_2}} L\left\{\sum_{k=0}^{\infty} \underline{C}_k\right\} + \frac{\phi}{s^{\alpha_2}} L\left\{\sum_{k=0}^{\infty} \underline{A}_k\right\},$$

$$L\{\textstyle\sum_{k=0}^{\infty} M_k\} = \frac{M}{s} - \frac{\upsilon}{s^{\alpha_3}} L\{\textstyle\sum_{k=0}^{\infty} M_k\} + \frac{\mu}{s^{\alpha_3}} L\{\textstyle\sum_{k=0}^{\infty} \underline{B}_k\}. \quad (11)$$

Matching the two sides of (11) yields the following iterative algorithm:

$$L\{\eta_0\} = \frac{\eta}{s} + \frac{\omega}{s^{\alpha_1+1}} L\{\eta_1\} = -\frac{\sigma}{s^{\alpha_1}} L\{\eta_0\} - \frac{\varepsilon}{s^{\alpha_1}} L\{\underline{A}_0\} - \frac{\xi}{s^{\alpha_1}} L\{\underline{B}_0\} L\{\eta_2\} = -\frac{\sigma}{s^{\alpha_1}} L\{\eta_1\} - \frac{\varepsilon}{s^{\alpha_1}} L\{\underline{A}_1\} - \frac{\xi}{s^{\alpha_1}} L\{\underline{B}_1\} \ldots L\{\eta_{k+1}\} = -\frac{\sigma}{s^{\alpha_1}} L\{\eta_k\} - \frac{\varepsilon}{s^{\alpha_1}} L\{\underline{A}_k\} - \frac{\xi}{s^{\alpha_1}} L\{\underline{B}_k\}$$

Next,

$$L\left\{\sum_{k=0}^{\infty} F_k\right\} = \frac{F}{s} - \frac{\gamma}{s^{\alpha_2}} L\left\{\sum_{k=0}^{\infty} F_k\right\} - \frac{\lambda}{s^{\alpha_2}} L\left\{\sum_{k=0}^{\infty} \underline{C}_k\right\} + \frac{\phi}{s^{\alpha_2}} L\left\{\sum_{k=0}^{\infty} \underline{A}_k\right\}, L\{F_0\} = \frac{F}{s}, L\{F_1\} = -\frac{\gamma}{s^{\alpha_2}} L\{F_0\} - \frac{\lambda}{s^{\alpha_2}} L\{\underline{C}_0\} + \frac{\phi}{s^{\alpha_2}} L\{\underline{A}_0\}, L\{F_2\} = -\frac{\gamma}{s^{\alpha_2}} L\{F_1\} - \frac{\lambda}{s^{\alpha_2}} L\{\underline{C}_1\} + \frac{\phi}{s^{\alpha_2}} L\{\underline{A}_1\}, \ldots L\{F_{k+1}\} = -\frac{\gamma}{s^{\alpha_2}} L\{F_k\} - \frac{\lambda}{s^{\alpha_2}} L\{\underline{C}_k\} + \frac{\phi}{s^{\alpha_2}} L\{\underline{A}_k\},$$

and

$$L\left\{\sum_{k=0}^{\infty} M_k\right\} = \frac{\underline{M}}{s} - \frac{\upsilon}{s^{\alpha_3}} L\left\{\sum_{k=0}^{\infty} M_k\right\} + \frac{\mu}{s^{\alpha_3}} L\left\{\sum_{k=0}^{\infty} \underline{B}_k\right\} L\{M_0\} = \frac{\underline{M}}{s} L\{M_1\}$$
$$= -\frac{\upsilon}{s^{\alpha_3}} L\{M_0\} + \frac{\mu}{s^{\alpha_3}} L\{\underline{B}_0\} L\{M_2\}$$
$$= -\frac{\upsilon}{s^{\alpha_3}} L\{M_1\} + \frac{\mu}{s^{\alpha_3}} L\{\underline{B}_1\} \ldots L\{M_{k+1}\}$$
$$= -\frac{\upsilon}{s^{\alpha_3}} L\{M_k\} + \frac{\mu}{s^{\alpha_3}} L\{\underline{B}_k\}$$

This provides the recurrence formula to estimate the $\eta_0, \eta_1, \eta_2, \eta_3, \ldots$, $F_0, F_1, F_2, F_3, \ldots$, and $M_0, M_1, M_2, M_3, \ldots$.

4. Approximation Solution

In this section, we apply the LADM to the fish farm model. Here $\eta(0) = \bar{\eta} = 1, \mathrm{F}(0) = \bar{\mathrm{F}} = 1$ and $\mathrm{M}(0) = \overline{\mathrm{M}} = 1$ for the three-component model, and we set $\xi = 0.1, \sigma = 0.3, \upsilon = 0.4, \omega = 0.25$ [20] few first approximations for X(t), Y(t) and Z(t) are calculated and presented below: According to LADM, we obtain

$\varepsilon = 0.2,\ \ \phi = 0.8,\ \lambda = 0.4,\ \gamma = 0.1,\ \ \mu = 1.5,$

The approximate solutions for $\eta(t), F(t), and\ M(t)$ are given by the series:

$$\eta(t) = \sum_{k=0}^{\infty} \eta_k(t), \quad \mathrm{F}(t) = \sum_{k=0}^{\infty} \mathrm{F}_k(t), \quad \mathrm{M}(t) = \sum_{k=0}^{\infty} \mathrm{M}_k(t).$$

Now the first approximation is obtained by using the initial conditions:

$$\eta_0(t) = 1 + \frac{0.25t^{\alpha_1}}{\Gamma(\alpha_1 + 1)},$$
$$\mathrm{F}_0(t) = 1,$$
$$\mathrm{M}_0(t) = 1.$$

Hence the Adomian polynomials are

$$\underline{A}_0 = F_0\eta_0 = 1 + \frac{0.25t^{\alpha_1}}{\Gamma(\alpha_1 + 1)} \underline{B}_0 = M_0\eta_0 = 1 + \frac{0.25t^{\alpha_1}}{\Gamma(\alpha_1 + 1)} \underline{C}_0 = {F_0}^2 = 1$$

The second approximation is obtained as:

$$\eta_1(t) = -0.6\left(\frac{t^{\alpha_1}}{\Gamma(\alpha_1+1)} + \frac{0.25t^{2\alpha_1}}{\Gamma(2\alpha_1+1)}\right), F_1(t)$$
$$= -0.5\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)} + 0.8\left(\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)} + \frac{0.25t^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1+\alpha_2+1)}\right), M_1(t)$$
$$= -0.4\frac{t^{\alpha_3}}{\Gamma(\alpha_3+1)} + 1.5\left(\frac{t^{\alpha_3}}{\Gamma(\alpha_3+1)} + \frac{0.25t^{\alpha_1+\alpha_3}}{\Gamma(\alpha_1+\alpha_3+1)}\right).$$

Hence the series approximate solution is given by

$$\eta_0(t) = \eta_0(t) + \eta_1(t) + \ldots$$
$$= 1 + \frac{0.25t^{\alpha_1}}{\Gamma(\alpha_1+1)} - 0.6\left(\frac{t^{\alpha_1}}{\Gamma(\alpha_1+1)} + \frac{0.25t^{2\alpha_1}}{\Gamma(2\alpha_1+1)}\right) + \ldots \quad (11)$$

$$F(t) = F_0(t) + F_1(t) + \ldots$$
$$= 1 - 0.5\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)} + 0.8\left(\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)} + \frac{0.25t^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1+\alpha_2+1)}\right) + \ldots \quad (12)$$

$$M_1(t) = M_0(t) + M_1(t)$$
$$= 1 - 0.4\frac{t^{\alpha_3}}{\Gamma(\alpha_3+1)} + 1.5\left(\frac{t^{\alpha_3}}{\Gamma(\alpha_3+1)} + \frac{0.25t^{\alpha_1+\alpha_3}}{\Gamma(\alpha_1+\alpha_3+1)}\right) + \ldots \quad (13)$$

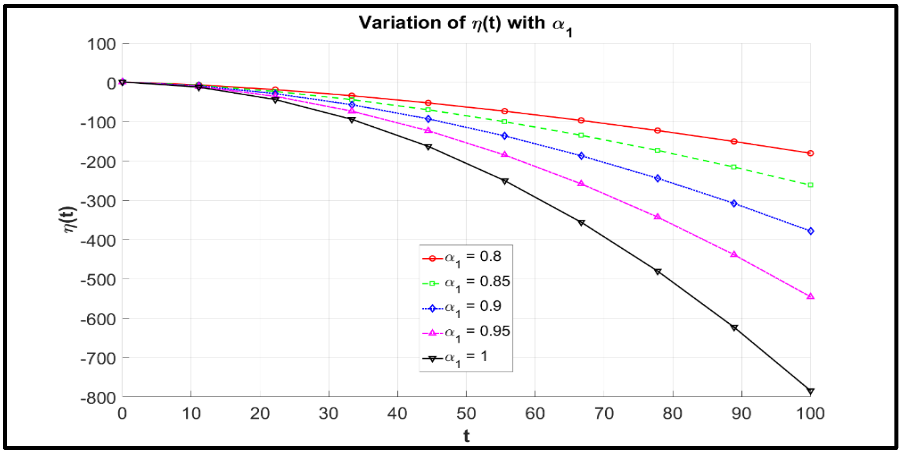
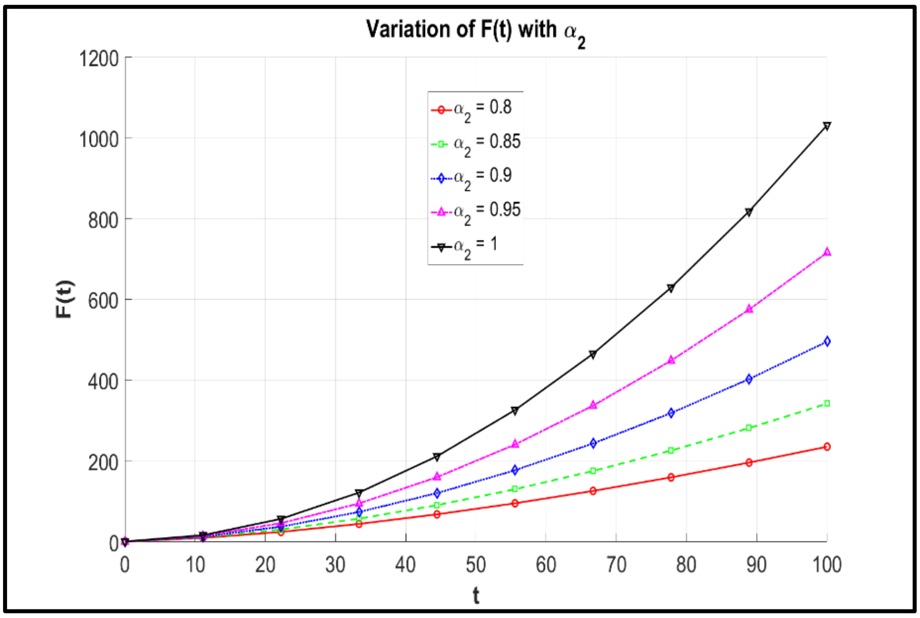


Figure: 1 Progress of $\eta(t)$ the densities of nutrient for different fractional orders

Figure: 2 Progress of $F(t)$ the densities of fish population for different fractional orders



Variation of M(t) with α_3

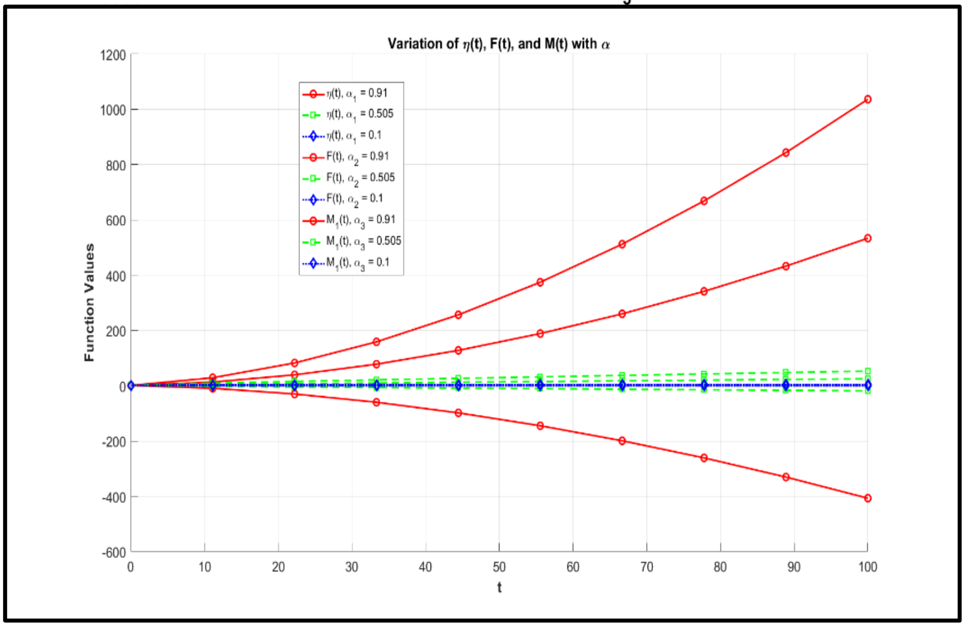


Figure: 3 Progress of $M(t)$ the densities of Muscles population for different fractional orders

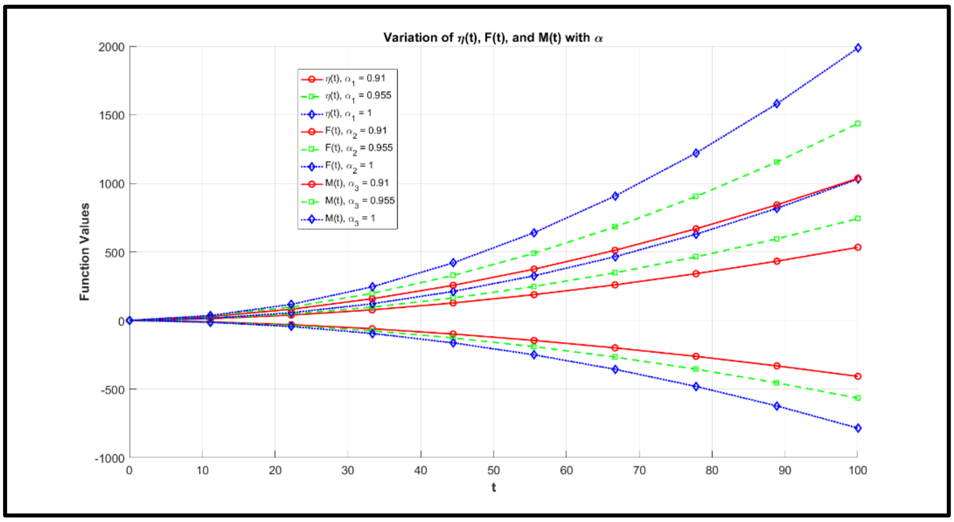


Figure 4 Progression of densities of nutrient $\eta(t)$ fish population $F(t)$ and mussel biomass M (t) at time t for Mixed fractional orders $0.9 < \alpha_1, \alpha_2, \alpha_3 \leq 1$.

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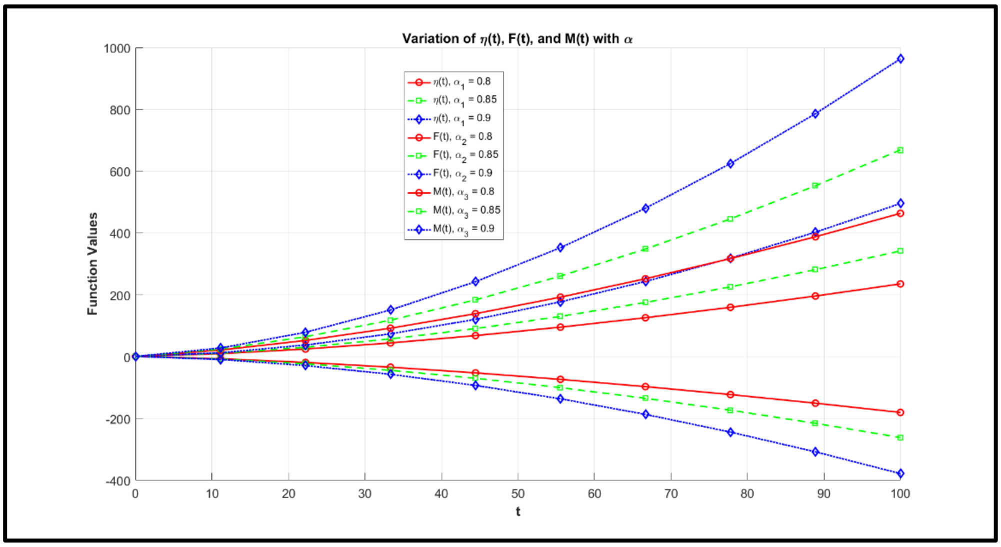

Figure: 5 Progression of densities of nutrient $\eta(t)$ fish population $F(t)$ and mussel biomass M (t) at time t for Mixed fractional orders $0.8 \leq \alpha_1, \alpha_2, \alpha_3 \leq 0.9$

4. Result: This study investigates a **fractional-order fish farm model** involving three interacting components: **nutrients (η), fish (𝔽), and mussels (𝔐)**, governed by a system of fractional differential equations. The model accounts for nutrient uptake, competition, and ecological interactions, providing a more generalized framework than classical integer-order models.

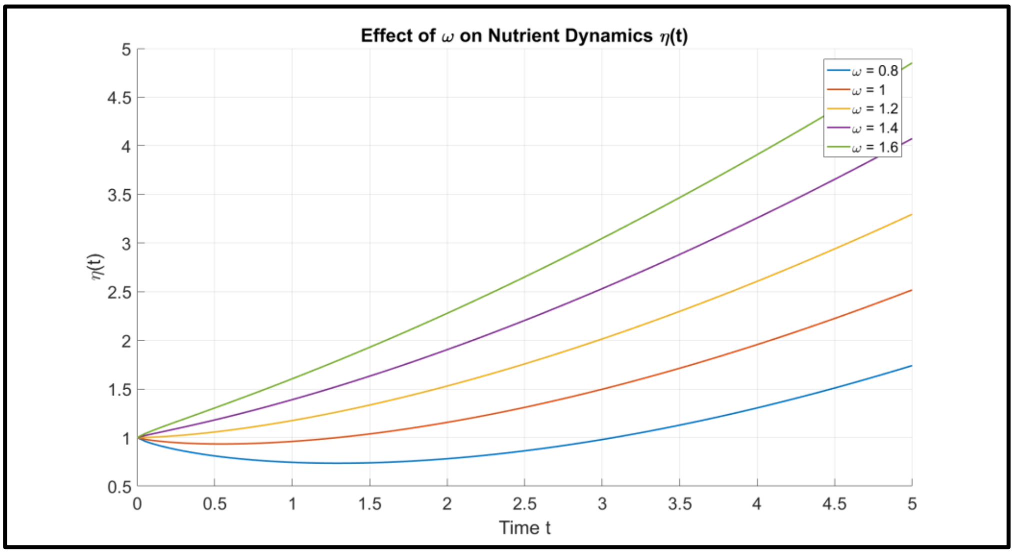

Figure 6: Effect ω of on Nutrient Dynamics for the fractional order $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$

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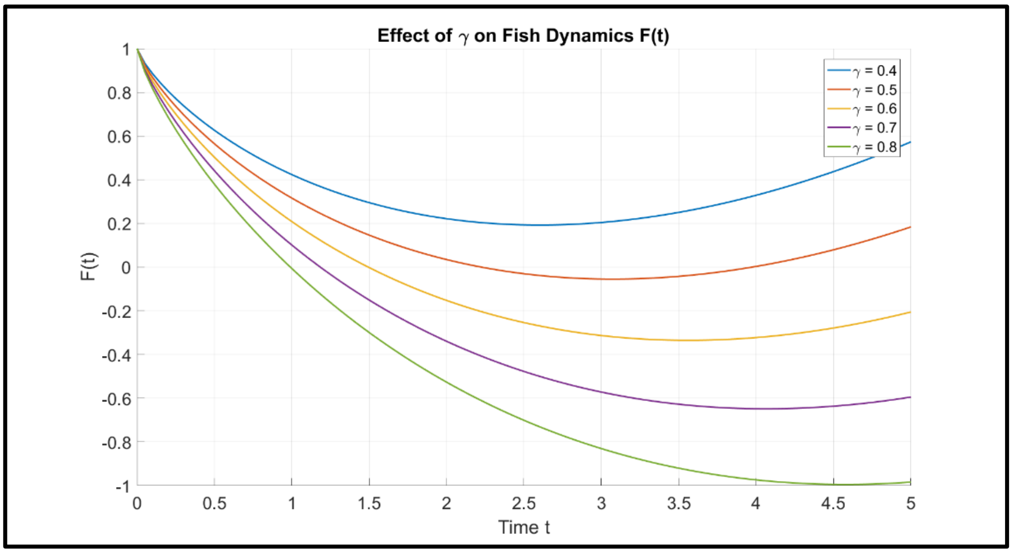

Figure 7: Effect γ of on Fish Dynamics for the fractional order $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$

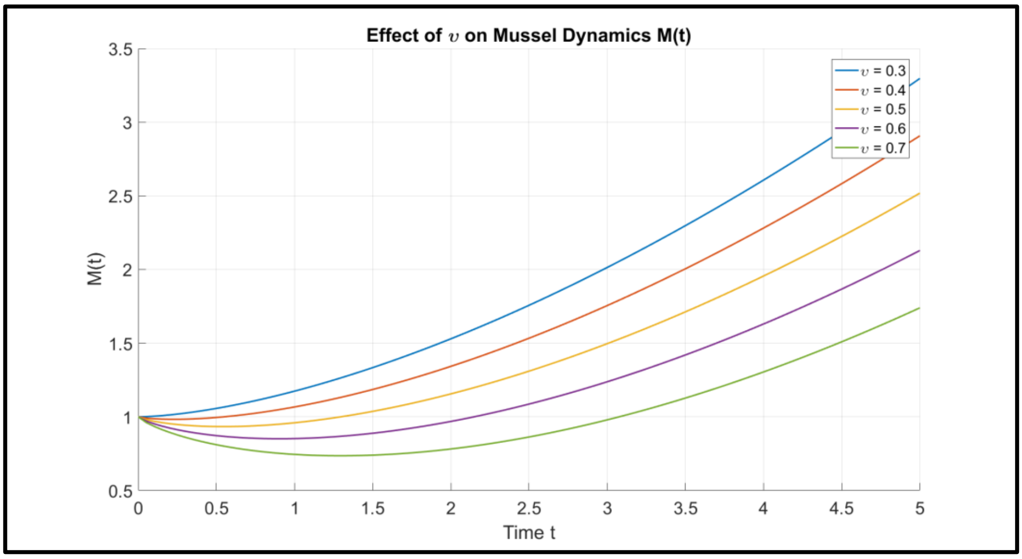

Figure 8: Effect υ of on Mussel Dynamics for the fractional order $\alpha_1 = \alpha_2 = \alpha_3 = 0.8$

The **Laplace-Adomian Decomposition Method (LADM)** effectively approximates the solutions, revealing the impact of fractional-order parameters **(α_1, α_2, α_3)** on system dynamics. The key observations include:

- **Nutrient Dynamics ($\eta(t)$)**: The external food input **(ω)** and nutrient uptake rates **(σ, ε, ξ)** influence the availability of nutrients. Higher **ω** can lead to eutrophication, altering ecosystem stability.
- **Fish Population ($\mathbb{F}(t)$)**: The fish biomass dynamics depend on mortality **(γ)**, competition **(λ)**, and nutrient assimilation **(ϕ)**. The results show that fractional effects delay population stabilization, capturing more realistic growth patterns.

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- **Mussel Population ($\mathfrak{M}(t)$):** Mussels exhibit nonlinear growth behaviour influenced by mortality **(υ)** and nutrient conversion efficiency **(μ)**. The fractional-order model provides a more flexible representation of population persistence and environmental adaptation.

5. Conclusion:

This study successfully applies the Laplace-Adomian Decomposition Method (LADM) to solve a fractional-order fish farm model, demonstrating the method's efficiency in handling nonlinear ecological systems. The results confirm that fractional calculus offers a more accurate and flexible approach to modelling nutrient flow and species interactions. The model highlights the impact of external food input, competition, and nutrient conversion on ecosystem sustainability. Future research may explore parameter estimation from real-world data and extend the model to incorporate environmental stressors such as climate change and pollution.

6. References

[1] G. Adomian, A review of the decomposition method and some recent results for nonlinear equations, Comput. Math. Appl, 21 (5) (1991) 101-127.

[2] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer, Dordrecht, 1994.

[3] A. Ardito, S. de Gregorio, L. Lamberti, P. Ricciardi, Dynamics of a lake ecosystem, in L.M. Ricciardi (Ed.), Biomathematics and Related Computational Problems, (1988) 111-119.

[4] D. Bahuguna, A. Ujlayan and D.N. Pandeya, A comparative study of numerical methods for solving an integro-differential equation, Comput. Math. Appl, 57 (2009) 1485-1493.

[5] J. C. Butcher, Numerical methods for ordinary differential equations, Second edition, John Wiley & Sons, 2008.

[6] S. N. Elgazery, Numerical solution for the Falkner-Skan equation, Chaos, Solitons and Fractals, 35 (2008) 738-746.

[7] J. Fadaei and M. M. Moghadam, Numerical Solution of Systems of Integral Differential Equations by Using Modified Laplace-Adomian Decomposition Method, World Applied Sciences Journal 19 (2012) 1818-1822.

[8] N. H. Gazi, Dynamics of populations in fish farm: Analysis of stability and direction of Hopf-bifurcating periodic oscillation, Appl. Math Modell, 36 (2012) 2118-2127.

[9] N. H. Gazi, S.R. Khan and C.G. Chakrabarti, Integration of mussel in fish farm: Mathematical model and analysis, Nonlinear Analysis: Hybrid Systems, 3 (2009)74-86.

[10] M. Hussain and M. Khan, Modified Laplace decomposition method, Appl. Math. Sci, 36 (2010) 1769-1783.

[11] Y. Khan, An effective modification of the Laplace decomposition method for nonlinear equations, International Journal of Nonlinear Sciences and Numerical Simulation, 10 (2009) 1373-1376.

[12] S. A. Khuri, A Laplace decomposition algorithm applied to class of nonlinear differential equations, J Math. Appl, 4 (2001) 141-155.

[13] S. A. Khuri, A new approach to Bratu's problem, Appl. Math. Comput, 147 (2004) 131-136.

[14] S. Momani and V.S. Erturk, Solutions of non-linear oscillators by the modified differential transform method, Comput. Math. Appl, 55 (2008) 833-842.

[15] M. Y. Ongun, The Laplace Adomian Decomposition Method for solving a model for HIV infection of CD4+T cells, Math, Comp. Appl, 55 (2008) 833-842.

[15] M. Y. Ongun, The Laplace Adomian Decomposition Method for solving a model for HIV infection of CD4+T cells, Math. Comp. Modell, 53 (2011) 597-603.

[16] H. Padé, Sur la représentation approchée d'une fonction par des fractions rationnelles, Ann. Sci. Éc. Norm. Sup., 9 (1892) 1-93.

[17] F. Saei, F. Dastmalchi, D. Misagh, Y. Zahiri, M. Mahmoudi, V. Salehian, and N. Rafati Maleki, New Application of Laplace Decomposition Algorithm For Quadrtic Riccati Differential Equation by Using Adomian's Polynomials, Life Science Journal, 10 (2013) 3s.

[18] E. Yusufoglu, Numerical solution of Duffing equation by the Laplace decomposition algorithm, Appl. Math. Comput, 177 (2006) 572-580.

[19]S. Handibag & B. D., Karande, Laplace substitution method for solving partial differential equations involving mixed partial derivatives. International Journal of Pure and Applied Mathematics, (2012), 78, 973–979.

[20] M., Sambath, and K., Balachandran, Laplace Adomian decomposition method for solving a

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Vol 19 Issue 2 May 2026

fish farm model. Nonautonomous Dynamical Systems, (2016) 3(1), pp.104-111.

[21] Adomian, G. (1994). Solving Frontier Problems of Physics: The Decomposition Method. Kluwer.

[22] Wazwaz, A. M. (2009). Partial Differential Equations and Solitary Waves Theory. Springer.

[23] Khuri, S. A. (2001). A Laplace decomposition algorithm applied to a class of nonlinear differential equations. Journal of Applied Mathematics, 1(4), 141–155.